AD-A090 053 STANFORD UNIV CA DEPT OF COMPUTER SCIENCE F/G 12/1
NUMERICAL LINEAR ALGEBRA.(U)
SEP 80 D L BOLEY, G H GOLUB, P VAN DOOREN AFOSR-79-0094
UNCLASSIFIED AFOSR-TR-80-0874 NL

1 OF 1
AD A
090053

END
DATE
FILMED
11-80
DTIC

AFOSR-TR- 80-0874

LEVEL II

(4)

AD A090053

INTERIM SCIENTIFIC REPORT

Air Force Office of Scientific Research Grant AFOSR-79-0094

Period: 1 February 1979 through 31 March 1980

Title of Research: NUMERICAL LINEAR ALGEBRA

Principal Investigators: Gene H. Golub

James H. Wilkinson

Research Personnel: Paul Van Dooren

Daniel L. Boley

DTIC
ELECTE
OCT 7 1980
S D
A

Computer Science Department
Stanford University
Stanford, California 94305

DDC FILE COPY

Approved for public release;
distribution unlimited.

80 10 2 044

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER AFOSR-TR-80-0874	2. GOVT ACCESSION NO. AD-A090 053	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) NUMERICAL LINEAR ALGEBRA		5. TYPE OF REPORT & PERIOD COVERED Interim ~~Scientific~~ Report, 2/1/79-3/31/80
		6. PERFORMING ORG. REPORT NUMBER 1 Feb 79-31 Mar 80
7. AUTHOR(s) Daniel L. Boley, Gene H. Golub, Paul Van Dooren, James H. Wilkinson		8. CONTRACT OR GRANT NUMBER(s) AFOSR-79-0094
9. PERFORMING ORGANIZATION NAME AND ADDRESS Department of Computer Science, Stanford University, Stanford, CA 94305		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 2304/A3 61102F
11. CONTROLLING OFFICE NAME AND ADDRESS Air Force Office of Scientific Research/NM, Bolling Air Force BAse, DC 20332		12. REPORT DATE 9/8/80
		13. NUMBER OF PAGES 7
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) 8 Sep 80 9		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

APPROVED FOR PUBLIC RELEASE; DISTRIBUTION UNLIMITED

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Research in this program has concentrated on the generalized eigenvalue problem and its natural extension to the computation of the associated canonical form. Furthermore, there has been an extensive effort to study the matrix equation arising in control engineering such as controllability observability decomposition and the solution of the Riccati equations. In particular, error bounds for the computed eigenvalues and eigenvectors of the generalized eigenvalue problem have been devised. (cont)

(Cont)

In addition, a numerically stable algorithm has been developed for computing the orthonormal bases for any deflating subspace of a regular pencil. A method has been developed to obtain any desired ordering of eigenvalues in the quasi-triangular forms.

ABSTRACT

Research in this program has concentrated on the generalized eigenvalue problem and its natural extension to the computation of the associated canonical form. Furthermore, there has been an extensive effort to study the matrix equation arising in control engineering such as controllability observability decomposition and the solution of the Riccati equations. In particular, error bounds for the computed eigenvalues and eigenvectors of the generalized eigenvalue problem have been devised. In addition, a numerically stable algorithm has been developed for computing the orthonormal bases for any deflating subspace of a regular pencil. A method has been developed to obtain any desired ordering of eigenvalues in the quasi-triangular forms.

RESEARCH OBJECTIVES AND STATUS OF THE RESEARCH

James H. Wilkinson

I. The calculation of error bounds for computed eigenvalues and eigenvectors

In practice it is rare for the complete eigensystem of a large matrix to be required. Commonly, attention is focussed on a few key eigenvalues and eigenvectors. An algorithm is therefore desirable for deriving error bounds for a single eigenpair (i.e. value and vector) without requiring information on the remainder of the system. A method for deriving rigorous error bounds for such a pair λ and x was developed and is about to be published in Numerische Mathematik (a preliminary version appeared as a Stanford report [1]). A pleasing feature is that in the process of deriving the bounds an improved eigenpair was derived and the bound was

AIR FORCE OFFICE OF SCIENTIFIC RESEARCH (AFSC)
NOTICE OF TRANSMITTAL TO DDC
This technical report has been reviewed and is approved for public release IAW AFR 190-12 (7b).
Distribution is unlimited.
A. D. BLOSE
Technical Information Officer

for this eigenpair.

A less pleasing feature was that the bound (and indeed the true performance) was limited by the condition number of a certain matrix C . This matrix C is ill-conditioned when λ is close to a double root. Multiple roots are not necessarily ill-conditioned and hence this is an inherent weakness of the method.

This weakness has been overcome by determining generators of the invariant subspace associated with clusters of eigenvalues. This material is to be presented as in invited lecture at an international symposium at Zürich in honour of H. Rutishauser and the paper [2] will be published in the proceedings of that symposium.

Extensions of the algorithm which make it computationally more efficient have been developed by J. Dongarra of Argonne National Laboratory in his Ph.D. thesis. A joint paper on these extensions by Dongarra, Moler and Wilkinson is in preparation. All algorithms apply both to the standard and the generalized problem.

II. The determination of the distance of a matrix A from the nearest defective matrix and the corresponding problem for $Ax = Bx$

This is a problem of considerable interest to control engineers. Early work in connection with the standard problem was done by Kahan, Ruhe and Wilkinson. The techniques developed by them have been greatly improved and extended to the generalized problem (the more important case). In the course of this work, rather general results were derived which should be of considerable value in the backward error analysis of eigenvalue algorithms. This work is to be presented as an invited lecture at an international meeting to be held in Winnipeg in October and the paper [3] will be published in the proceedings.

III. The double QR algorithm of Frances

This algorithm reduces a general real matrix A to quasi-triangular matrix T , which has two by two matrices on the diagonal corresponding to complex conjugate pairs of roots. A disadvantage of the algorithm is that the eigenvalues are not ordered in any systematic way and orderings are often required by control engineers.

Algorithms are required for interchanging the order of

(a) Adjacent diagonal elements

(b) A two by two block and an adjacent diagonal

(c) Two adjacent two by two blocks.

The first is almost trivial (it was first derived by Ruhe). The other problems have been referred to briefly by Golub and Wilkinson but no solutions were given. Methods for achieving the interchanges based on orthogonal similarities have been developed and will be the subject of a forthcoming paper when they have been fully tested.

References

[1] H. J. Symm & J. H. Wilkinson, Realistic Error Bounds for a Simple Eigenvalue and its Associated Eigenvector. Stanford Computer Science Report No. STAN-CS-80-787. Accepted for publication in Numer. Math.

[2] H. J. Symm and J. H. Wilkinson, Error Bounds for Computed Invariant Subspaces. To be presented as an invited paper at an International Symposium to be held at ETH Zürich in honor of H. Rutishauser, October 1980.

[3] H. J. Symm and J. H. Wilkinson, Almost Defective Matrices. To be presented an an invited paper at an International Symposium to be held at the University of Manitoba, October 1980.

Paul Van Dooren

I. Computation of zeros of linear multivariable systems

Several algorithms have been proposed in the literature for the computation of the zeros of a linear system described by a state-space model $\{\lambda I - A, B, C, D\}$. We have devised a new algorithm; the new approach handles both nonsquare and/or degenerate systems without difficulties whereas earlier methods either fail or require special tratment for these cases. The method is also backward stable in a rigorous sense.

II. A generalized eigenvalue approach for solving Riccati equations

A numerically stable algorithm has been derived to compute orthonormal bases for an deflating subspace of a regular pencil $\lambda B - A$. The method is based on an update of the QZ algorithm, in order to obtain any desired ordering of eigenvalues in the quasi-triangular forms constructed by this algorithm.

The computation of deflating subspaces with specified spectrum is of crucial importance in solving Riccati equations arising in linear system theory.

Daniel Boley

I. Computing the controllability-observability decomposition of a linear time-invariant dynamic system, a numerical approach

Various numerical properties are involved in computing the complete Controllability-Observability (Kalman) Decomposition for a linear time-invariant dynamic system, of the form

$$\dot{x} = Ax + Bu$$

$$y = Cx$$

where A,B,C are matrices, and u, x, y are vector functions of time. In particular, we are investigating the numerical stability, the cost and the particular advantages of several algorithms.

PUBLICATIONS

Boley, D. L., "On Kalman's procedure for the computation of the controllable/observable canonical form," Proceedings of the 19th IEEE Conference on Decision and Control, Albuquerque, NM, December 1980.

Van Dooren, P., "A generalized eigenvalue approach for solving Riccati equations," Report NA-80-02, Numerical Analysis Project, Stanford University, July 1980. Submitted to the SIAM Journal on Scientific and Statistical Computing. Workshop Numer. Meth. in Aut. Contr., 9/80.

Van Dooren, P., and Emami-Naeni, A., "Computation of zeros of linear multivariable systems," Report NA-80-03, Numerical Analysis Project, Stanford University, July 1980, accepted for IEEE Conf. Dec. & Contr.

Wilkinson, J. H. and Symm, H. J., "Realistic error bounds for a simple eigenvalue and its associated eigenvector," Report STAN-CS-80-787, Stanford University, Computer Science Department, January 1980. To appear in Numerische Mathematik.

PERSONNEL ASSOCIATED WITH RESEARCH EFFORT

Professor Gene H. Golub

Professor James H. Wilkinson

Dr. Paul Van Dooren, Post-doctoral Research Fellow, Philips Research Laboratory, Brussels

Daniel Boley, Graduate Student. Expected date of Ph.D. degree, December 1980. Dissertation title: Inverse Eigenvalue Problems and the Use of Advance Control Structures in Numerical Analysis

INTERACTIONS

Invited Papers

Daniel Boley 19th IEEE Conference on Decision and Control, Albuquerque, NM, December 1980.

Paul Van Dooren Workshop on Numerical Methods in Automatic Control, September 1980.

Paul Van Dooren 19th IEEE Conference on Decision and Control, Albuquerque, NM, December 1980.

J. H. Wilkinson International Symposium in Honour of H. Rutishauser to be held at ETH-Zentrum, Zürich, October 1980.

J. H. Wilkinson International Symposium to be held at the University of Manitoba, October 1980.

Consultative and advisory functions

Gene H. Golub Visit to the Defense Mapping Agency, June 1980.